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NUMERICAL SOLUTION TO DISCRETE SYSTEMS OF BLOCK STRUCTURE WITH BOUNDARY CONDITIONS UNSHARED BETWEEN BLOCKS

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Abstract. In the work, the numerical approach to solution to discrete dynamical systems of large dimension, having a block structure with boundary conditions unshared between blocks have been proposed. Formulas for carrying over boundary conditions are obtained and results of numerical solution to the problem, illustrating effectiveness of the suggested method, are provided.

Keywords: discrete dynamical systems, decomposition of a complex object, non-separated boundary conditions, the method of transferring boundary conditions.

AMS Subject Classification: 49J15, 49J35.

1. Introduction

The article is devoted to the numerical solution to discrete dynamical systems of large dimension, having a block structure with boundary conditions unshared between blocks.

Direct usage of methods of carrying over boundary conditions is not efficient due to the block structure of conditions that allows, as for many other classes of problems, significantly accelerate their solution.

We note that a lot of encountered in practice mathematical models of discrete dynamic models of complex processes have been obtained by using the decomposition of complex objects into simpler subobjects with known mathematical models or subobjects, for which mathematical models could be easily constructed ([1,7,9,10,12]). The decomposition may be carry out respect to spatial and/or temporal variables, and it is worth noting that the decomposition of a complex object is held so that the intermediate states of subobjects are not influenced each other, i.e. are independent, and the connection between subobjects is implemented only through input and output states of subobjects ([4,5,7]). Moreover, in practice subobjects are usually associated with an arbitrary but small number of other subobjects, and consequently conditions determining the relationship between subobjects are characterized by a weakly filled Jacobi matrices ([9,10]).

In this paper we propose a numerical approach to solving discrete systems of block structure with weak and arbitrary connections between subsystems. The approach is based on the idea of methods of carrying over boundary conditions

([1,2,5,6]). The idea of article [6], where the numerical solution of a system of independent three point discrete equations with non-separated boundary conditions was considered , has been further developed in this article for the case of discrete systems of block structure with boundary conditions unshared between blocks. Not only were the formulas for implementation of transferring conditions obtained, also the results of numerical experiments are provided. We have considered the solution to the problem arising when applying methods of finite difference approximation to equations systems with partial derivatives of hyperbolic type, describing the movement of fluids in the pipeline of complex structure.

2. Statement of the problem

Consider the system of equations describing complex discrete process (the object) consisting of mutually independent L discrete subprocesses (subobjects), each of them is described by a system of linear algebraic equations

$$y^{\nu i+1} = A^{\nu i} y^{\nu i} + B^{\nu i}, \; i = 1,..., N_\nu - 1, \; y^{\nu i} \in R^{n_\nu}, \; \nu = 1,\dots,L. \tag{1}$$

Here $y^{\nu i} = \left(y_1^{\nu i},\dots,y_{n_\nu}^{\nu i}\right)^*$ is n_ν -dimensional vector defining the state of the $\nu-$th process in $i-$ th discrete instant of time; $A^{\nu i}$ and $B^{\nu i}$ are accordingly n_ν -dimensional square matrix and vector; $rang\ A^{\nu i} = n_\nu$, $i = 1,..., N_\nu$; N_ν is duration of the $\nu-$ th process; $\nu = 1,\dots,L$; * is the sign of transposition.

We introduce the notation

$$n = \sum_{\nu=1}^{L} n_\nu, \qquad M = \sum_{\nu=1}^{L} n_\nu N_\nu, \qquad y^{\nu k} = \left(y_1^{\nu k},\dots,y_{n_\nu}^{\nu k}\right)^* \in R^{n_\nu},$$

$$y^1 = \left(y^{1,1}, y^{2,1},\dots,y^{L,1}\right)^* \in R^n, \qquad y^N = \left(y^{1,N_1}, y^{2,N_2},\dots,y^{L,N_L}\right)^* \in R^n.$$

Here M is the overall dimension of the whole system consisting of subsystems (1), $y^1 \in R^n$ and $y^N \in R^n$ are accordingly the states of all subprocesses in the initial and final (individual for each subprocess) instants of time.

Considered subprocesses are connected through initial and final states in the shape of unshared boundary conditions, written in the form:

$$Gy^1 + Qy^N = R, \tag{2}$$

where $G = \left(\left(g^{ij}\right)\right)$, $Q = \left(\left(q^{ij}\right)\right)$ are matrices with dimension $n \times n$, $R = \left(R^1,\dots,R^n\right)^*$ is given $n-$dimensional vector.

We assume that the rank of the augmented matrix (G,Q) is equal to n, i.e. $rang\,(G,Q) = n$, and in general the system of equations (1), (2) has a solution, and the only one.

Conditions (2) are written in vector form, which will be used subsequently

$$\sum_{s=1}^{L} g^{is*} y^{s1} + \sum_{s=1}^{L} q^{is*} y^{sN_s} = R^i \text{ , } i = 1,..., n, \tag{3}$$

where $g^{is} = \left(g_1^{is},\ldots,g_{n_s}^{is}\right)^*$, $q^{is} = \left(q_1^{is},\ldots,q_{n_s}^{is}\right)^*$.

The relations (1), (2) represent mathematical models for many complex objects, processes functioning discretely with lumped or distributed parameters ([3-8]). At the same time for their mathematical modeling was applyed the decomposition method upon temporal and / or spatial variables, i.e., partitioning the entire object into separate subobjects, which function independently from each other, and connection between them is implemented through their input and output states, i.e. by the conditions (2).

Boundary value problems described by systems of differential equations with ordinary or partial derivatives, for solving which grid methods were applyed, can also be converted to considered problems of the form (1), (2) ([1-6, 10-12]). In this case systems of equations themselves consist of separate independent subsystems, connected only by means of initial and/or boundary conditions. In particular, the problem of calculating the unsteady motion of fluid, gas in pipeline transportation networks with complex, loopback structure is reduced to a system (1), (2) The motion process itself on each linear section is described by a hyperbolic system of two partial differential equations of first order ([3-5,7,8]).

Mathematical models of many real large objects with complex structure are characterized by the following peculiarities: 1) a large number of subobjects L; 2) small dimensionality of the subobjects state vector n_ν, $\nu = 1,\ldots,L$; 3) long duration of functioning N_ν, $\nu = 1,\ldots,L$; 4) weak and arbitrary interconnections between the subobjects, i.e. weak and arbitrary filling of matrices G, Q and vectors g^{is}, q^{is}, $s = 1,\ldots,L$.

Features 1), 3) for real objects lead to the fact that the order of algebraic system (1), (3), being equal to M, may exceed several thousand and tens of thousands, that doesn't allow to use known numerical methods of solving algebraic equations systems for their decision. Feature 4) leads to unshared boundary conditions, making it necessary to use methods of carrying over boundary condition.

The peculiarity 2) allows easily to get relations which are equivalent to (1) , but in reverse order of calculating:

$$y^{\nu,i} = \overline{A}^{\nu,i+1} y^{\nu,i+1} + \overline{B}^{\nu,i+1}, \quad i = N_\nu - 1,...,1 \text{ , } \quad \nu = 1,\ldots,L, \tag{4}$$

$$\overline{A}^{s,i} = \left(A^{s,i+1}\right)^{-1}, \quad \overline{B}^{s,i} = \left(A^{s,i+1}\right)^{-1} B^{s,i}, \quad i = 1,..., N_s - 1, \quad s = 1,..., L.$$

The aim of this work is to develop an efficient numerical method for solving the system of discrete equations (1) with unseparated boundary conditions (2), (3) taking into account peculiarities mentioned above. The method is based on the analogue of method of carrying over (transfer) conditions and is reduced to solving series of specially built discrete Cauchy problems with respect to separate subsystems of the system (1).

3. Numerical solution to the problem

The proposed approach to solving the considered problem is based on transferring boundary conditions (3) to one end: to the left or right. This means that relations (2) or (3) will be replaced by equivalent relations, in which the vector y^1 will be missing when transferring conditions to the right end:

$$\tilde{Q}y^N = \tilde{R} \tag{5}$$

or

$$\sum_{s=1}^{L} \tilde{q}^{is*} y^{sN_s} = \tilde{R}^i,\ i = 1,..., n, \tag{6}$$

and the vector y^N will be missing when transferring conditions to the left end

$$\tilde{G}y^1 = \tilde{R} \tag{7}$$

or

$$\sum_{s=1}^{L} \tilde{g}^{is*} y^{s1} = \tilde{R}^i,\ i = 1,..., n\,. \tag{8}$$

After transferring conditions to one end the systems (5), (6) or (7), (8) will be obtained, which represent systems of n algebraic equations with n unknowns: y^1 or y^N. After solving these systems and defining y^1 or y^N, the solution of overall task is achieved by carrying out simple calculations using explicit recurrent formulas (Cauchy problems) with respect to separate subsystems of discrete equations (1) (while carrying over to the left) or subsystems (4) (while porting conditions to the right).

Selecting the direction of carrying over the conditions (2), (3) depends on the degree of filling matrices G and Q. Namely, when matrix G is less filled in comparison with the matrix Q, then conditions need to be ported to the right, and conversely, if matrix G is filled stronger than Q, the conditions should be ported to the left. This rule will become apparent after the following description of the procedure of carrying over.

Transferring conditions (2), more precisely (3), will be implemented separately for each $i-$th condition, $i = 1,\dots,n.$

Thus, let us consider an arbitrary $i-$th condition in (3), that will take the form (6) after carrying over to the right side, where the $\tilde{q}^{is*}$, $\tilde{R}^i$ are yet unknown new coefficient values. Obtaining conditions in the form (6) will be carried out in stages.

Assume that not of all vector coefficients g^{ij}, $j = 1,\ldots,L$ are equal to zero, otherwise there would be no need to move the $i-$th condition to the right, since this condition involves only the values y^N. Let the first nonzero coefficient is $g^{i\nu}$, that is $g^{i\nu} \neq 0_{n_\nu}$, $g^{ij} = 0_{n_j}$, $j < \nu$ ($0_{n_\nu} - n_\nu$-dimensional vector, all of whose components are equal to 0).

In this case, one could say that $n_\nu -$dimensional vectors $\alpha^k \in R^{n_\nu}$ and scalars β^k, $k = 1,..., N_\nu$ perform carrying over the $i-$th condition (3) in respect to the $\nu-$th vector of unknowns $y^{\nu k}$ to the right, if for all vectors $y^{\nu k}$ satisfying to the $\nu-$th subsystem (1) following equalities hold

$$\alpha^{k*} y^{\nu k} + \sum_{s=\nu+1}^{L} g^{is*} y^{s1} + \sum_{s=1}^{L} q^{is*} y^{sN_s} = \beta^k, \qquad k = 1,..., N_\nu . \tag{9}$$

It is clear that under $k = 1$ must be performed equalities:

$$\alpha^1 = g^{i\nu}, \qquad \beta^1 = R^i . \tag{10}$$

Vectors α^k and scalars β^k, $k = 1,..., N_\nu$ satisfying (9), (10), will be called transfer coefficients. Substituting in (10) the values of transfer coefficients under $k = N_\nu$ we obtain a new condition

$$\sum_{s=\nu+1}^{L} g^{is*} y^{s1} + \sum_{s=1}^{L} \tilde{q}^{is*} y^{sN_s} = \tilde{R}^i ,$$

in which following notation introduced

$$\tilde{q}^{i\nu} = q^{i\nu} + \alpha^{N_\nu}, \ \tilde{q}^{ij} = q^{ij}, \quad j = 1,..., L, \ j \neq \nu, \ \tilde{R}^i = \beta^{N_\nu} .$$

Transfer coefficients α^k, β^k, that carry over the condition (3) to the right, can be determined in different ways. One of them is offered in the following theorem.

Theorem 1. Let the $n_\nu -$dimensional vectors α^k and scalars β^k, $k = 1,..., N_\nu$ are defined by the following recurrence relations (discrete Cauchy problems):

$$\alpha^{k+1} = \overline{A}^{\nu k} \alpha^k, \qquad \alpha^1 = g^{i\nu}, \ k = 1,..., N_\nu ,$$

$$\beta^{k+1} = \beta^k + \alpha^{k+1*} B^{\nu k}, \quad \beta^1 = R^i, \ k = 1,..., N_\nu . \tag{11}$$

Then α^k, β^k are the transfer coefficients, carrying over the $i-$th condition in (3) to the right respect to the $y^\nu -$ th decision of the $\nu-$th subsystem (1).

Proof. According to (10) under $k = 1$ condition (9) is equivalent to $i-$th condition in (3).

Assume that α^k, β^k and α^{k+1}, β^{k+1} under $k > 1$ satisfy condition of carrying over the $i-$th condition regarding the $y^\nu -$ decision of the $\nu-$th subsystem (1).

Consequently, this implies:

$$\alpha^{k*}y^{\nu k}+\left[\sum_{j=\nu+1}^{L}g^{ij*}y^{j1}+\sum_{j=1}^{L}q^{ij*}y^{jN_j}\right]=\beta^k,\ k=1,\ldots,N_\nu, \tag{12}$$

$$\alpha^{k+1*}y^{\nu,k+1}+\left[\sum_{j=\nu+1}^{L}g^{ij*}y^{j1}+\sum_{j=1}^{L}q^{ij*}y^{jN_j}\right]=\beta^{k+1}\ . \tag{13}$$

We take into account the $\nu-$th subsystem of (1) in (13):

$$\alpha^{k+1*}\left[A^{\nu k}y^{\nu k}+B^{\nu k}\right]+\left[\sum_{j=\nu+1}^{L}g^{ij*}y^{j1}+\sum_{j=1}^{L}q^{ij*}y^{jN_j}\right]=\beta^{k+1}\ .$$

After subtracting equation (12) from this equation and subsequent grouping we obtain:

$$[\alpha^{k+1*}A^{\nu k}-\alpha^{k*}]y^{\nu k}+[-\beta^{k+1}+\beta^k+\alpha^{k+1*}B^{\nu k}]=0$$

Given that this equation should hold for all possible solutions of the $\nu-$th subsystem (1), we will require from coefficients α^k, β^k, α^{k+1}, β^{k+1} the equality to zero of expressions in square brackets

Taking into account (4), we obtain the necessary relations for transfer coefficients in the form (11). After completing the procedure for replacing the values of $\nu-$th vector $y^{\nu 1}$ in the $i-$th condition by value $y^{\nu N_\nu}$ with a new coefficient $\tilde{q}^{\nu N_\nu}$, we obtain a new condition equivalent to the previous one.

In this condition there is no value of $y^{\nu 1}$. Next, proceed to the next non-zero coefficient g^{ij}, $j>\nu$, until we get the condition $g^{ij}=0_{n_j}$, $j=1,\ldots,L$. This means that $i-$th condition has been completely transferred to the right. Further all the specified procedure is implemented for ($i+1$)- th condition. If $(i+1)>n$, then all conditions (3) have been transferred to the right, and as a result there have been obtained conditions of the form (5) or (6), equivalent to conditions (3).

Conditions (5) and (6) represent a system of n linear algebraic equations relative to $n-$dimensional vector y^N. Solving this system yields the vector y^N, and further the desired solution $y=\left(y^1,\ldots,y^N\right)^*$ of the problem is determined by (1).

Similar to the above procedure of transferring conditions to the right , a successive transfer of conditions to the left is carried out in order to get conditions (7) or (8), being equivalent to the conditions (3).

Suppose that in $i-$th condition among vectors q^{ij}, $j=1,\ldots,L$, the first non-zero vector is $q^{i\nu}$, i.e. $q^{ij}=0_{n_j}$, $j<\nu$, $q^{i\nu}\neq 0_{n_j}$.

We will say that $n_\nu -$dimensional vectors α^k and scalars β^k, $k=1,\dots,N_\nu$ perform carrying over the $i-$th condition (3) in respect to the vectors $y^{\nu k}$ being solutions of the $\nu-$th subsystem (1) to the left, if following equalities hold:

$$\alpha^{k*}y^{\nu k}+\sum_{s=1}^{L}g^{is*}y^{s1}+\sum_{s=\nu+1}^{L}q^{is*}y^{sN_s}=\beta^k,\ k=1,\dots,N_\nu. \quad (14)$$

$$\alpha^{N_\nu}=q^{i\nu},\ \beta^{N_\nu}=R^i\ . \quad (15)$$

It is obvious that (14) under $k=N_\nu$ coincides with the $i-$th condition (3).

If α^k, β^k, $k=1,\dots,N_\nu$ are sweep coefficients, then from equality (14) under $k=1$ we get a new condition

$$\sum_{s=1}^{L}\tilde{g}^{is*}y^{s1}+\sum_{s=\nu+1}^{L}q^{is*}y^{sN_s}=\tilde{R}^i,$$

that is equivalent to the $i-$th condition, in which introduced the notation:

$$\tilde{g}^{i\nu}=g^{i\nu}+\alpha^1,\ \ \tilde{g}^{ij}=g^{ij},\quad j=1,\dots,N_\nu,\ j\neq\nu.$$

This condition differs from the condition (3) so that its $i-$th part doesn't contain the summand with $y^{\nu N_\nu}$. Further, this procedure is repeated until there is at least one coefficient q^{is} different from zero. After that, carrying over is performed for the next ($i+1$)-th condition if $i+1\le n$. Left transfer coefficients that carry over $i-$th condition to the left, can be determined from the following theorem.

Theorem 2. Let the $n_\nu -$dimensional vectors α^k and scalars β^k, $k=1,\dots,N_\nu$ are defined by the following recurrence relations (discrete Cauchy problems):

$$\alpha^k=A^{\nu k*}\alpha^{k+1},\quad \alpha^{N_\nu}=q^{iN_\nu},\ k=N_\nu-1,N_\nu-2,\dots,1,,$$

$$\beta^k=\beta^{k+1}-B^{\nu k*}\alpha^{k+1},\quad \beta^{N_\nu}=R^i,\ k=N_\nu-1,N_\nu-2,\dots,1. \quad (16)$$

Then α^k, β^k are the sweep coefficients for carrying over the $i-$th condition in (3) to the right regarding the $y^\nu -$ th decision of the $\nu-$th subsystem (1).
The proof is similar to the above proof of Theorem 1.

The very process of converting all the conditions (2), (3) to the (5), (6) by carrying over values $y^{\nu N_\nu}$ to the left is similar to the process described above for carrying over conditions to the right. Completion of the carrying over process leads to a system of n algebraic equations regarding $n-$dimensional vector y^1. Further after solving this system we hold a recurrent calculation of the desired subsystems solutions $y^{\nu k}$ of system (1) from left to right, $k=1,\dots,N_\nu$, $\nu=1,\dots,L$.

4. Results of numerical experiments

Consider the following system of discrete equations, consisting of five subsystems ($L=5,\ n_\nu=2,\ N_\nu=201,\ \nu=1,...,5$):

$$y_1^{1,k+1}=y_1^{1,k}+0{,}005\,y_2^{1,k},\qquad y_2^{1,k+1}=y_1^{1,k}+1{,}01y_2^{1,k}-0{,}01e^{0{,}005k}+0{,}0099,$$

$k=1,\ldots,200$,

$$y_1^{2,k+1}=y_1^{2,k}+0{,}005\,y_2^{2,k},\qquad y_2^{2,k+1}=y_1^{2,k}+0{,}085\,y_2^{2,k}-0{,}0175\,e^{0{,}0025k}-$$

$$-0{,}005\cos(0{,}005k)-0.015\sin(0{,}005k)+0{,}045,$$

$$y_1^{3,k+1}=y_1^{3,k}+0{,}005\,y_2^{3,k},\qquad y_2^{3,k+1}=y_1^{3,k}+1{,}0025\,y_2^{3,k}-0{,}0025,\qquad (17)$$

$$y_1^{4,k+1}=y_1^{4,k}+0{,}005\,y_2^{4,k},\qquad y_2^{4,k+1}=y_1^{4,k}+1{,}005\,y_2^{4,k}-0{,}0025\,e^{0{,}0025k}-$$

$$-0{,}25\times10^{-4}k+0{,}005,$$

$$y_1^{5,k+1}=y_1^{5,k}+0{,}005\,y_2^{5,k},\qquad y_2^{5,k+1}=y_1^{5,k}+1{,}005\,y_2^{5,k}-0{,}0025\,e^{0{,}0025k}-$$

$$-1{,}25\times10^{-7}k^2+0{,}5\times10^{-4}k$$

with the following ten unshared conditions, including states in the initial and final moments:

$$y_1^{1,1}+y_1^{2,1}+y_1^{3,1}=0\,, \qquad (18)$$

$$y_2^{1,1}-y_2^{3,1}=0\,, \qquad (19)$$

$$y_2^{2,1}-y_2^{3,1}=0\,, \qquad (20)$$

$$y_1^{4,1}=4\,, \qquad (21)$$

$$y_1^{5,1}=-1\,, \qquad (22)$$

$$y_1^{3,N_3}+y_1^{4,N_4}+y_1^{5,N_5}=4\sqrt{e}-1/4\,, \qquad (23)$$

$$y_2^{3,N_3}-y_2^{5,N_5}=0\,, \qquad (24)$$

$$y_2^{4,N_4}-y_2^{5,N_5}=0\,, \qquad (25)$$

$$y_1^{1,N_1}=-1+3e\,, \qquad (26)$$

$$y_1^{2,N_2}=3-2\sqrt{e}+\sin(1)\,. \qquad (27)$$

It isn't difficult to verify that the solution of the problem (17)-(27) with accuracy up to 10^{-8} is represented by vectors, which components for $k=1,\ldots,201$ are defined as follows

$$y_1^{1,k}=0{,}25\times10^{-4}(k-1)^2+2e^{0{,}005(k-1)}-2,$$

$$y_2^{1,k}=0{,}01(k-1)+2e^{0{,}005(k-1)},$$

$$y_1^{2,k} = 0{,}015(k-1) - 2e^{0{,}0025(k-1)} + \cos(0{,}005(k-1)),\ y_2^{2,k} = 3 - e^{0{,}0025(k-1)} -$$

$$-\sin(0{,}005(k-1)),$$

$$y_1^{3,k} = 0{,}005(k-1) + 2e^{0{,}0025(k-1)} - 1, \qquad y_2^{3,k} = 1 + e^{0{,}0025(k-1)}, \tag{28}$$

$$y_1^{4,k} = 0{,}125\times 10^{-4}(k-1)^2 + 2e^{0{,}0025(k-1)} + 2,$$

$$y_2^{4,k} = 0{,}005(k-1) + e^{0{,}0025(k-1)},$$

$$y_1^{5,k} = 0{,}125\times 10^{-6}(k-1)^3 + 2e^{0{,}0025(k-1)} - 3,$$

$$y_2^{5,k} = 0{,}25\times 10^{-4}(k-1)^2 + e^{0{,}0025(k-1)}.$$

We note that the system of equations (17) and conditions (18) - (27) were obtained by simulating the finite-difference approximation of differential equation system with partial derivatives of hyperbolic type, describing the fluid motion in pipe network with structure shown in Fig. 1.

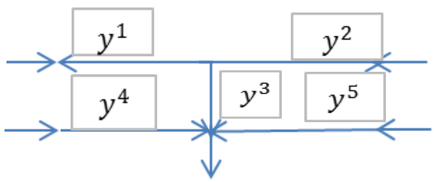

Fig 1. Conditional scheme of pipe network

The system (17) determines the motion mode only on the first layer at discrete points of the pipeline sections. All sections have equal lengths and are divided into 200 parts. Conditions (18) and (23) determine the material balance law at nodal points of the network, the conditions (19), (20), (24), (25) characterize conditions of flow continuity (equality of pressure values at the ends of sections adjacent to the node), conditions (21), (22), (26), (27) determine the operating modes of external sources (quantity of inflow and outflow of raw material by sources).

Taking into account an equal number of non-zero coefficients under $y^{\nu 1}$ and $y^{\nu N_\nu}$ in the conditions (18) - (27), the direction of carrying over conditions does not matter.

As a result of carrying over conditions (18) - (22) to the right the conditions were obtained in the form of an algebraic system (5), having a ten-dimensional matrix $\tilde{Q}$ presented in Table 1, and a right part -vector $\tilde{R}$ of the form:

$\tilde{R}^* = [0.00353 \quad 9.72566 \quad 0.0 \quad 0.0 \quad 0.005009 \quad -.07064 \quad 4.43656 \quad 0.24286 \quad 0.03244 \quad -.00821]$

Applying Gauss method with choosing the main member to solving the resulting system of equations, the vector was obtained

$y^N = (4.39986 \quad 4.43656 \quad 0.24032 \quad 0.24286 \quad 3.27706 \quad 3.29026 \quad 5.78862 \quad 5.80182 \quad 0.62037 \quad 0.63357\ )^*$.

By using this vector, the recurrent calculations were implemented to find $y^{\nu k}$, $k = \overline{201,1}$, $\nu = \overline{1,5}$, from subsystems of system (17). The accuracy of the obtained results did not exceed the value

$$\max_k \max_\nu \max_s \left|\Delta y_s^{\nu k}\right| \le 10^{-6}.$$

Table 1. Matrix $\tilde{Q}$ elements of the system 2.1.

$i \backslash j$	1	2	3	4	5	6	7	8	9	10
1	-0.5601	0.5638	-8.1103	8.1040	-1	0.9836	0	0	0	0
2	0.2331	-0.2331	0	0	-1	1	0	0	0	0
3	0	0	23.1173	-23.1173	-1	1	0	0	0	0
4	0	0	0	0	0	0	1	-0.9821	0	0
5	0	0	0	0	0	0	0	0	1	-0.9821
6	0	0	0	0	0	1	0	1	0	1
7	0	0	0	0	-1	1	0	0	1	-1
8	0	0	0	0	0	0	-1	1	1	-1
9	0	1	0	0	0	0	0	0	0	0
10	0	0	0	1	0	0	0	0	0	0

5. Conclusion

Numerical solution to discrete equation systems of large dimension, having a block structure with "weak" and arbitrary connections between subsystems have been considered in this work. Such systems have to be solved repeatedly while optimizing parameters of objects with complex structure or discretizing optimal control problems of processes described by equations with ordinary and partial derivatives. Schemes and corresponding formulas are proposed, based on idea of the method of carrying over boundary conditions, taking into account the peculiarities of the Jacobian of a system and 'weak' filling of the Jacobian matrix of connection conditions between subsystems.

The results of numerical experiments, obtained by solving problems arising during the calculation of unsteady motion mode of fluid in the pipe network with complex structure, to which have been applied implicit difference schemes of grid method, are provided.

References

1. Aida-zade K.R. Investigation of nonlinear optimization problems of network structure, Automation and Remote Control, No.2, 1990, pp.3-14, (in Russian).
2. Aida-zade K.R., Abdullaev V.M., On the Solution of Boundary Value Problems with Nonseparated Multipoint and Integral Conditions, Differential equations, Vol.49, No.9, 2013, pp.1114–1125.
3. Aida-zade K.R., Asadova D.A., The study of transients in pipelines, Automation and Remote Control, No.12, 2011, pp.156-172.
4. Akhmetzyanov A.V., Sal'nikov A.M., Spiridonov S.V., Multigrid balance models of unsteady flows in complex gas transportation systems, Managing large systems. Special Issue 30.1, Network model to manage, M.: ICS RAS, 2010, pp.230-251. (in Russian)
5. Aliev F.A., Zulfugarova R.T., Mutallimov M.M., Sweep algorithm for solving optimal control problems with a three-point boundary conditions, Journal of Automation and Information Sciences, 2008, No.4, pp.48-57 (in Russian).
6. Asadova J.A. Numerical solution of a system of independent three point discrete equations with non-separated boundary conditions, Proceesing of IAM, Vol.4, No.1, 2015, pp.58-69.
7. Efendiyev Y., Galvis J., Lazarov R.D, Margenov S., Ren J., Multiscale domain decomposition preconditioners for anisotropic high-contrast problems, Technical Report ISC-Preprint 2011-05, Institute for scientific Computation, 2011.
8. Guseynzade M.A., Yufin V.A., Unsteady flow of oil and gas in main pipelines. Moscow, Nedra, 1981, 232 p. (in Russian).
9. Juergen Geiser., Decomposition methods for differential equations: theory and applications, CRC Press 2010, 304p.
10. Krasovsky A.A., Decomposition and synthesis of suboptimal adaptive systems, Izv. USSR Academy of Sciences, Tech. Cyber., No.2, 1984, pp.157-165. (in Russian)
11. Samarsky A.A., Theory of difference schemes, M.: Nauka, 1983 (in Russian).
12. Tsurkov V.I., Decomposition in problems of large dimension, Moscow, Nauka, 1981, 352 p. (in Russian).

Ayrilmayan sərhəd şərtli blok strukturlu diskret sisteminin ədədi həlli

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İşdə ayrılmayan sərhəd şərtlərinə malik blok strukturlu diskret sisteminin həllinə ədədi yanaşma təklif edilmişdir. sərhəd şərtlərinin köçürülməsi üçün düsturlar alınmış, məsələnin ədədi həllinin nəticələri verilmişdir ki, bu da təklif edilən üsulun effektivliyini göstərir.

Açar sözlər: diskret dinamik sistemlər, mürəkkəb obyektin decompozisiyası, ayrılmayan sərhəd şərtləri, sərhəd şərtinin köçürülməsi üsulu.

Численное решение дискретных систем блочной структуры с неразделенными между блоками краевыми условиями

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РЕЗЮМЕ

В работе рассматривается численное решение дискретных динамических систем блочной структуры с неразделенными между блоками краевыми условиями. Получены формулы для осуществления переноса условий, приведены результаты численных экспериментов, иллюстрирующие эффективность предлагаемого подхода.

Ключевые слова: дискретные динамические системы, декомпозиция сложного объекта, неразделенные краевые условия, метод переноса краевых условий.